

Overview of Tessellation Techniques

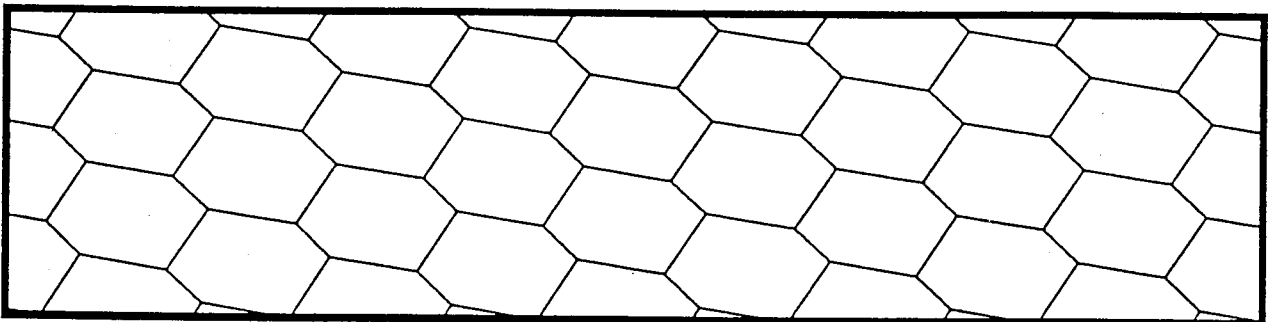
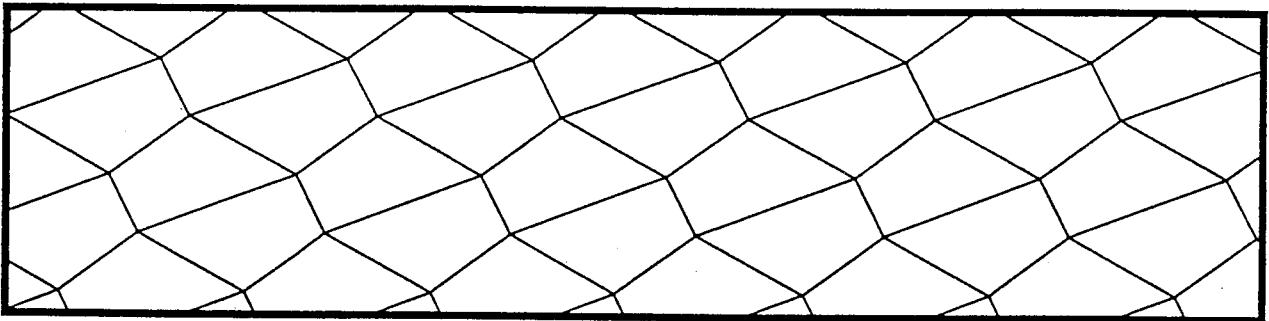
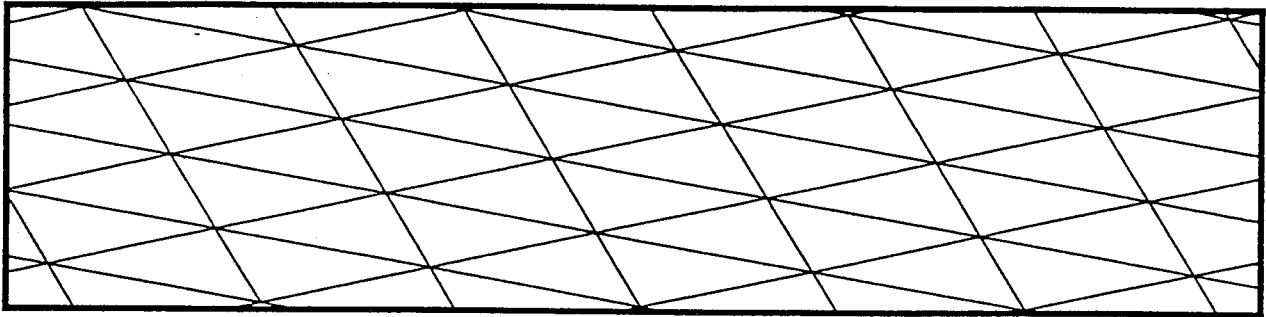
This overview is adapted from *Introduction to Tessellations* by Dale Seymour and Jill Britton (Palo Alto, CA: Dale Seymour Publications, 1989). For more details, please refer to that book.

Dutch graphic artist Maurits Cornelis Escher (1898–1972) created many perplexing tessellations. His preoccupation with tessellations developed after a 1936 visit to the Alhambra in Granada, Spain, an old Moorish palace decorated with mosaics in geometric patterns. Unlike the Moors, Escher did not restrict himself to abstract geometrical designs. Instead, he restricted himself to animate forms like the lizard we see in *Reptiles*, crawling out of Escher's two-dimensional sketch to explore the real world before rejoining his fellow reptiles in the interlocking design.



Reptiles by M. C. Escher (© 1943 M. C. Escher/Cordon Art, Baarn, Holland)

Escher's designs were plane tessellations. We define a tessellation of the plane as a pattern of shapes that fills the plane without any gaps or overlaps. The basis for any tessellating pattern is a grid of polygons—either triangles, quadrilaterals, or hexagons. Any triangle or quadrilateral } will tessellate the plane. In addition, any hexagon whose opposite sides are parallel and } congruent will tessellate. Examples of these tessellating grids are shown below.

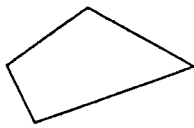


Various techniques can be used to transform triangles, quadrilaterals, or hexagons into animate shapes that tessellate the plane. These techniques involve transformations that we call * translation, rotation, and reflection. The following pages give a brief summary of these techniques, some used by student contest winners in creating their Escher-like drawings.

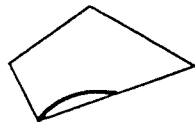
Tips for Beginners

Designing an Escher-like shape and drawing its tessellation is a time-consuming yet satisfying exercise. Following are some practical tips to help you get started.

A scalene quadrilateral allows you the greatest freedom and flexibility when you attempt your first design. However long its sides and whatever the size of its angles, a quadrilateral will tessellate the plane. If half of each of the sides is modified and the modifications are rotated 180° about their midpoints, the resulting shape will also tessellate the plane.



Step 1. Draw a scalene quadrilateral



Step 2. Modify half of any side



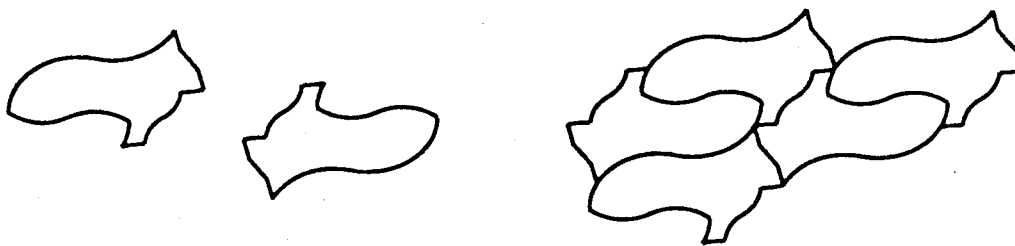
Step 3. Rotate modified line 180 degrees



Step 4. Repeat procedure with other 3 sides

Once your preliminary sketch is completed, you must prepare an accurate version of the shape for your tessellation. One simple and practical approach is to cut the original polygon shape from construction paper or lightweight cardboard, then cut appropriate "holes" and tape them on as corresponding "bumps" to represent your modifications. (In the case of glide reflection, you will need to flip the bump before taping.) **(OR DO IT ON A COMPUTER, LIKE US!)**

To create the tessellation, position your pattern on another sheet of paper, trace about its perimeter, and mark the location of interior details. By repositioning and tracing the pattern again and again, you will see the tessellation evolve before your eyes.

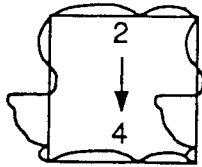
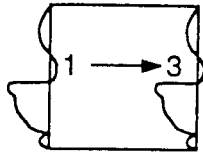


Modifying Polygons by Translation

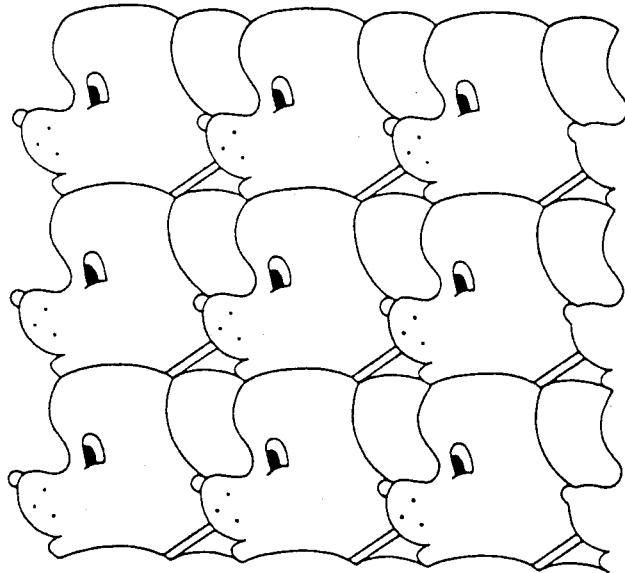
One way to create a tessellation design is to modify two sides of a parallelogram and translate those modifications to the opposite sides. A translation of a shape is a slide of that shape without rotation. The tessellating shape below is based on a square. The modification on side 1 is translated to side 3, and the modification on side 2 is translated to side 4.

Translation Summary:

Draw on one side, copy and paste on the opposite side. Same for all sides. Then erase original polygon and decorate inside.



Maximum Points: $\frac{20}{25}$ for a square or a parallelogram,
 $\frac{25}{25}$ for a hexagon.



Art by Tracy Steszyn

The example below demonstrates the result when we modify a parallelogram by translation. Adding details to the interior of the shape makes it more interesting.

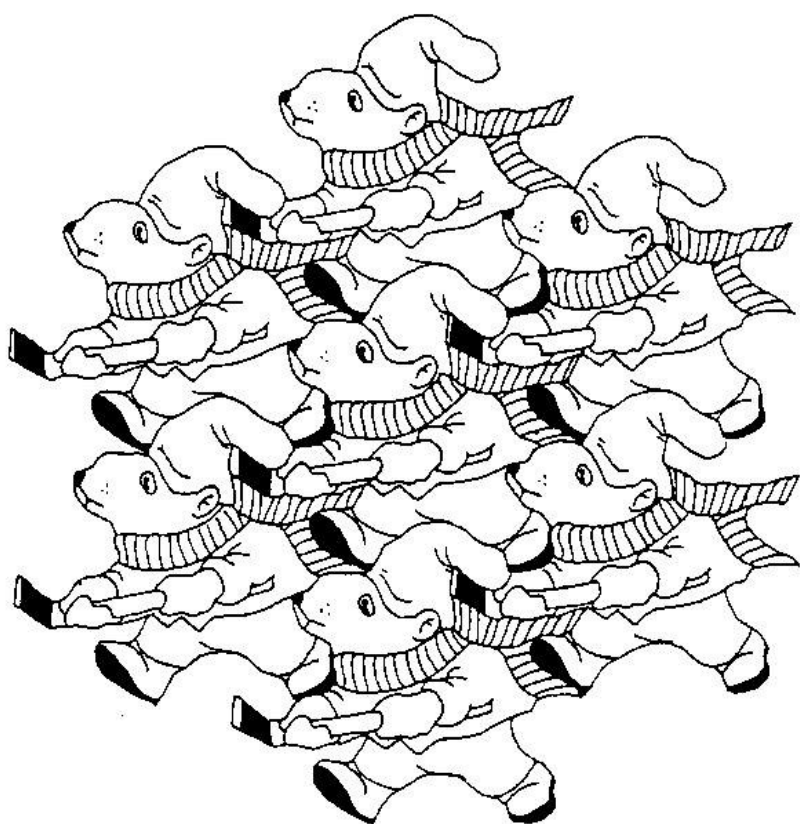
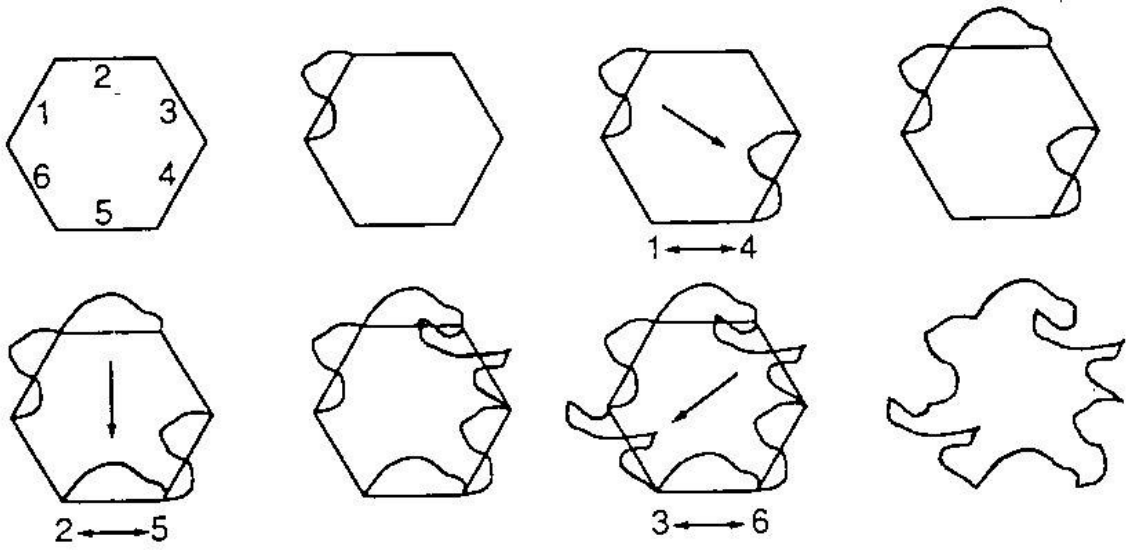


Art by Steve Dawson

For each of your tessellations, be sure to have (1) A creative title, (2) What your original shape was and (3) What kind of a tessellation it is. (For example, "Kevin Stomping Around The House"; Square; Translation)

For each of your tessellations, be sure to have (1) A creative title, (2) What your original shape was and (3) What kind of a tessellation it is. (For example, "Pirate Stomping Your Face"; Hexagon; Translation)

We can extend this method to include regular hexagons or, more generally, any hexagon having parallel and congruent sides. In hexagonal tessellations we have three sets of opposite sides to be modified. The following diagram demonstrates modifying a regular hexagon by translation. The tessellation, with details added, is shown below.



Art by Steve Dawson

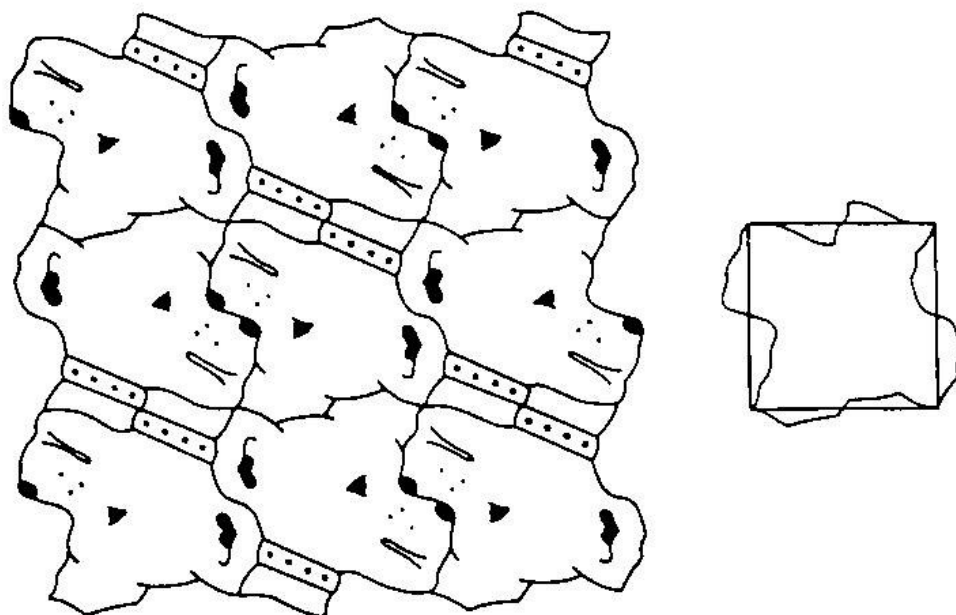
Translation Summary:

Draw on one side, copy and paste on the opposite side.

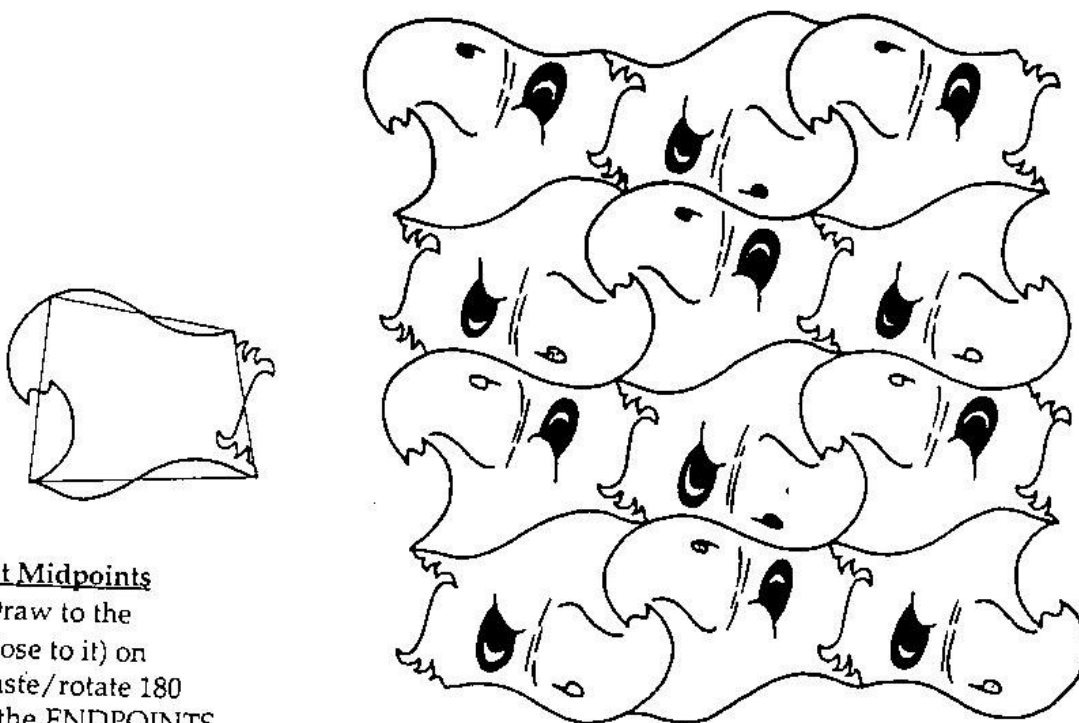
Maximum Points: $\frac{20}{25}$ for a square or a parallelogram, $\frac{25}{25}$ for a hexagon.

Modifying Polygons by Rotation about Midpoints of Sides of Sides

Any quadrilateral will tessellate. Given the enormous variety of quadrilaterals available to us, we can use rotation about midpoints of sides to create a wealth of Escher-like tessellating shapes. The examples below are based on a square and a scalene quadrilateral, each modified by rotation about midpoints of sides.



Art by Jill Britton



Art by Lyda Kobylansky

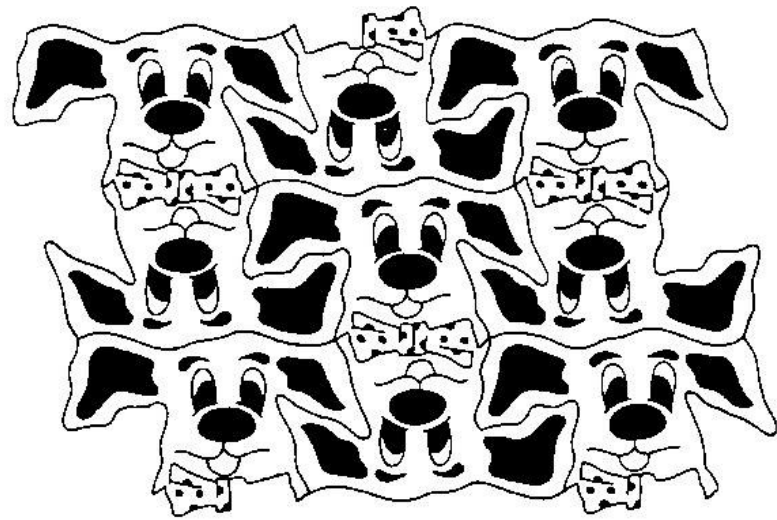
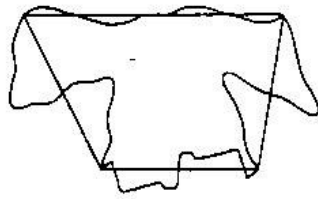
Rotation About Midpoints

Summary: Draw to the midpoint (or close to it) on one side, copy/paste/rotate 180...be sure to line up the ENDPOINTS when you paste! Repeat for all the other sides of your shape.

Maximum Points: $\frac{30}{25}$ for any shape.

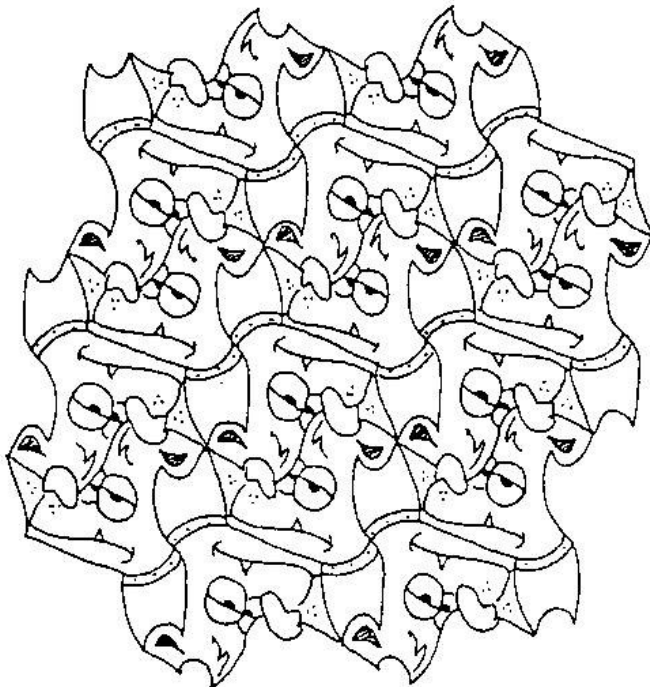
For each of your tessellations, be sure to have (1) A creative title, (2) What your original shape was and (3) What kind of a tessellation it is. (For example, "Teddy Bear Stomping Your Foot"; Quadrilateral; Rotation About Midpoints)

each of your tessellations, be sure to have (1) A creative title, (2) What original shape was and (3) What kind of a tessellation it is. (For example, *Stomping Out A Fire*; Trapezoid; Rotation About Midpoints)***

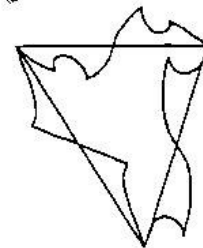


Art by Lyda Kobylansky

The example above shows a trapezoid modified by rotation about midpoints of sides. This same procedure works with any triangle. The below shape was created by modifying each half-side of a triangle and rotating the modification about the midpoint of the side.



Art by Steve Dawson



Rotation About Midpoints

Summary: Draw to the midpoint (or close to it) on one side, copy/paste/rotate 180 ...be sure to line up the ENDPOINTS when you paste! Repeat for all the other sides of your shape.

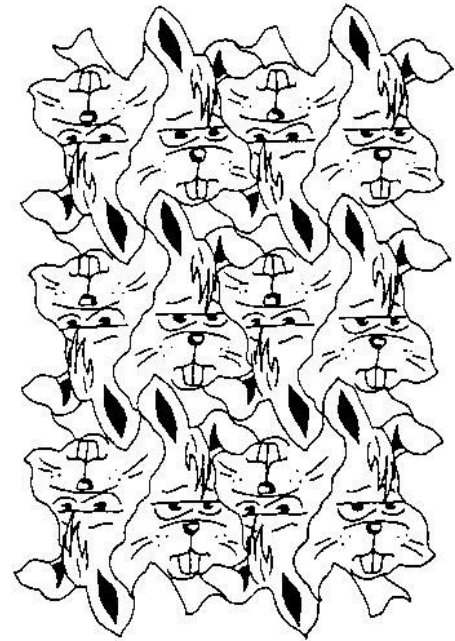
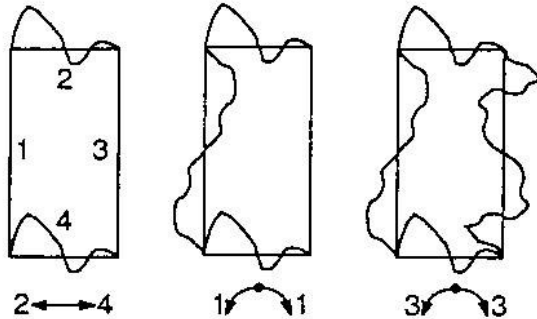
Maximum Points: $\frac{30}{25}$ for any shape.

Modifying Polygons by Translation and Rotation

Translation and Rotation

Summary: Draw on top, copy and paste on the bottom. Draw to midpoint on one side, copy/paste/rotate 180 and put on the other half (line up endpoints!) Repeat for other side (doesn't have to be the same as first side!)

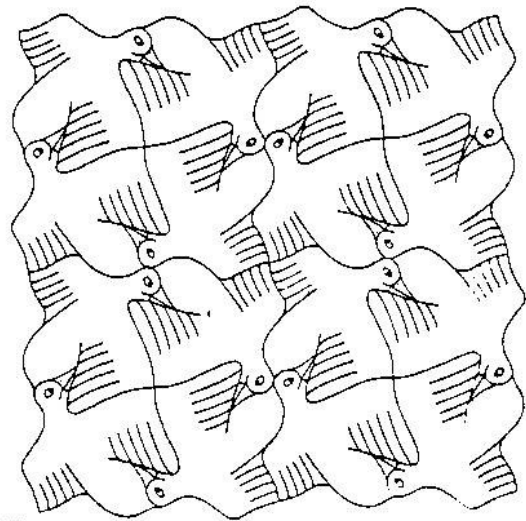
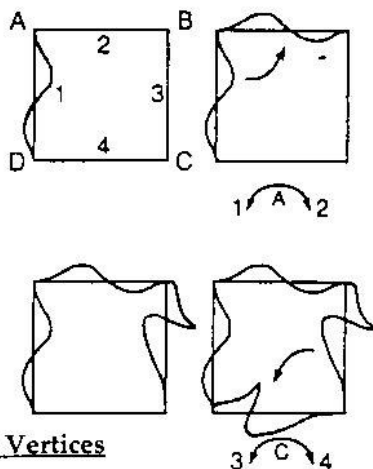
Maximum Points: $\frac{30}{25}$



Art by Steve Dawson

Modifying Polygons by Rotation about Vertices

Using another technique, the square below has been modified by rotating side 1 to side 3 and side 2 to side 4. To use this technique, the adjacent sides must be congruent. In this case we are rotating about a vertex of the polygon, not the midpoint of a side.



Art by Jill Britton

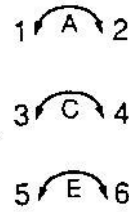
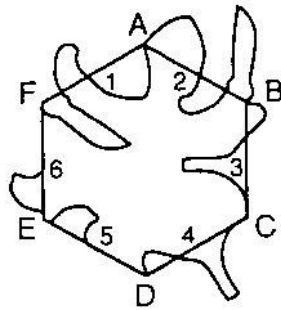
Rotation About Vertices

Summary: **Be sure you go right around the shape, don't skip around!** Draw on one side, copy/paste/rotate 90 and place on the adjacent side. Repeat for other sides.

Maximum Points: $\frac{25}{25}$ for square.

For each of your tessellations, be sure to have (1) A creative title, (2) What is your original shape was and (3) What kind of a tessellation it is. (For example, Easter Bunny Stomping On Eggs"; Rectangle; Translation & Rotation)***

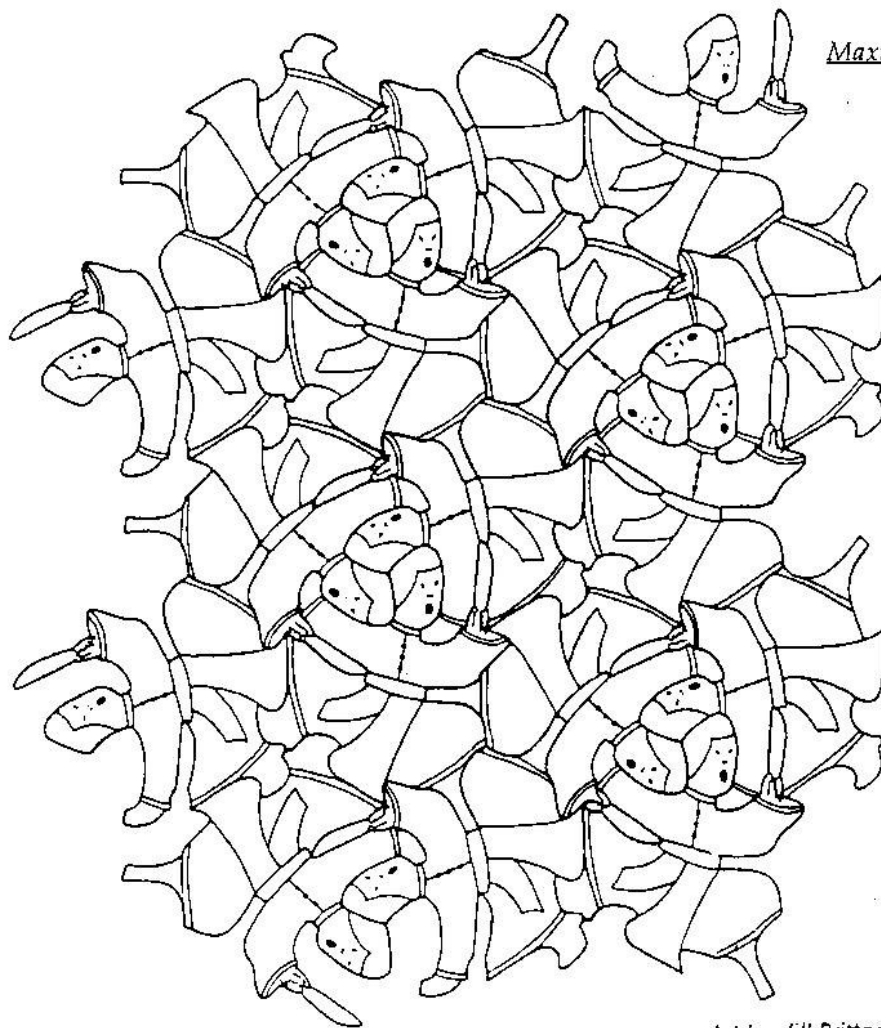
We can also create a tessellating shape by modifying a regular hexagon by rotation about vertices. In the example below, the resulting outline resembles a peg-legged pirate, even before we add interior details.



Rotation About Vertices

Summary: ****Be sure you go right around the shape, don't skip around!**** Draw on one side, copy/paste/rotate 120 and place on the adjacent side. Repeat for other sides. When you're done take your original shape, copy/paste/rotate 120... then paste and rotate 240. You now have a triplet and you can just paste that everywhere!

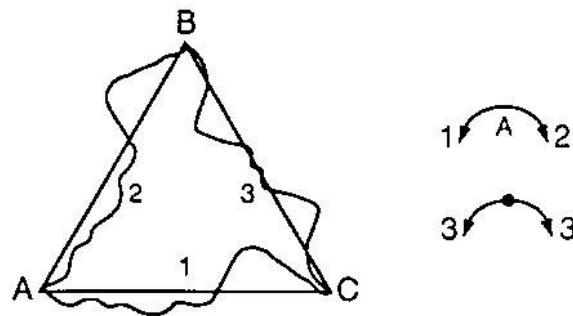
Maximum Points: $\frac{30}{25}$ for hexagon.



Art by Jill Britton

For each of your tessellations, be sure to have (1)A creative title, (2)What your original shape was and (3)What kind of a tessellation it is. (For example, "Santa Stomping Grapes"; Hexagon; Rotation Around Vertices)

If we start with an equilateral or isosceles triangle we can use this technique with two of the sides, rotating a modification about the vertex. To avoid being stuck with the third side as a straight line, we can modify half of this third side and rotate about the midpoint to create a tessellating shape. The design below demonstrates this technique.



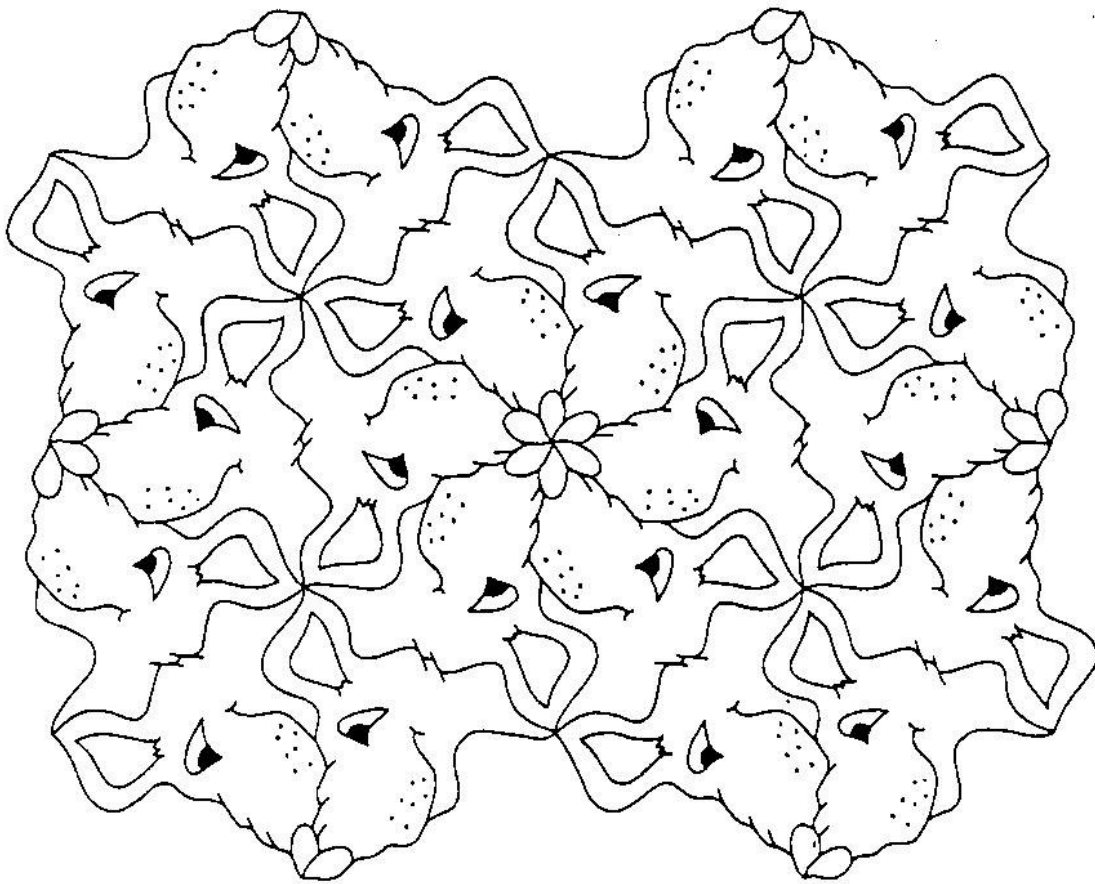
Rotation About Vertices

Using an Equil. Triangle:

Draw on the bottom, rotate 60 and paste on the left side. Then draw to the midpoint on the right side, copy/paste/rotate 180 on put on other half (line up endpoints!)

Now take original shape and rotate 60/120/180/240/300, and you'll have a 'sestet' (group of six) that you can paste everywhere!

Maximum Points: $\frac{30}{25}$



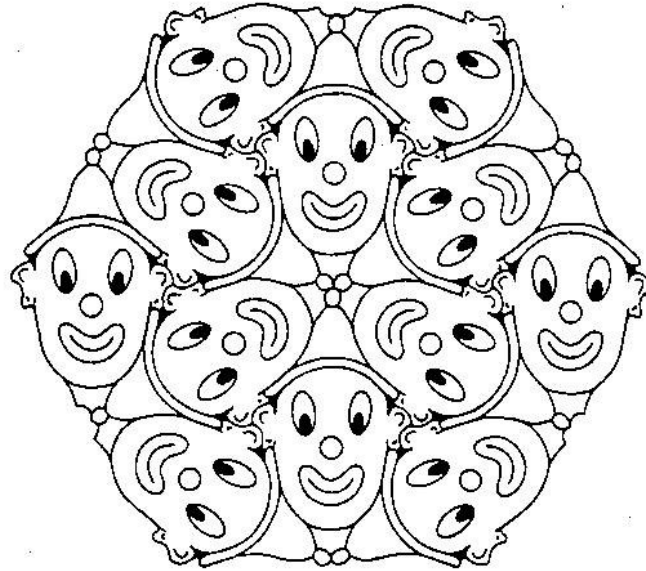
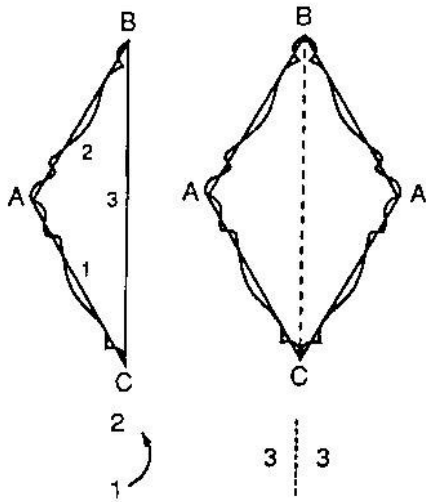
"Tisha," Art by Stephen Makris

Tessellation Winners: The Second Contest

For each of your tessellations, be sure to have (1) A creative title, (2) What your original shape was and (3) What kind of a tessellation it is. (For example, "My Dog Stomping My Cat"; Equilat. Triangle; Rotation Around Vertices)

Modifying Polygons by Reflection

Modification of tessellating polygons by reflection needs to involve either rotation or translation as well. An example that combines rotation and reflection in a special 120° isosceles triangle is shown below. Notice that the triangle is modified first by rotation at the vertex angle, then reflected to form a quadrilateral-based shape of a clown face with reflective symmetry.



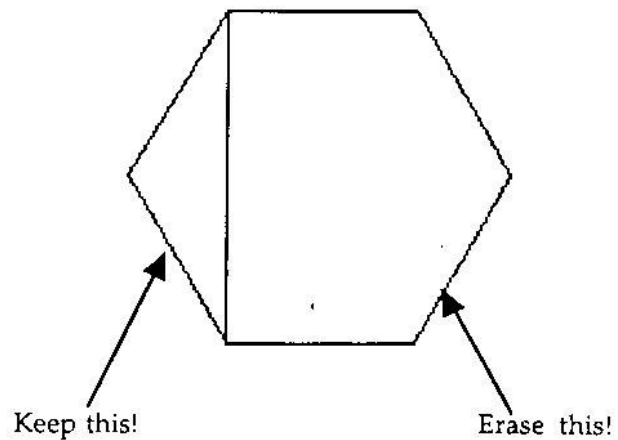
Art by Jill Britton

Reflection Summary:

To get your special 120 degree isosceles triangle, draw a hexagon (see right) and draw a line connecting the vertices shown. Then erase the rest of the hexagon.

To do your tessellation, draw on the bottom left side, copy/paste/rotate 120 and put it on the top left side. Then lasso that and copy/paste/flip horizontally and line up the two vertical lines. You're all set! Erase the original lines, decorate the inside and prepare to tessellate.

To tessellate: Copy/paste/rotate 120, then paste/rotate 240. You now have a triplet you can paste everywhere!

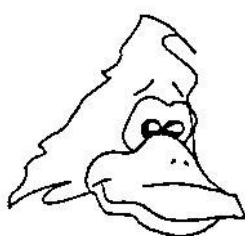
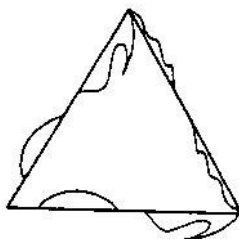


um Points: $\frac{30}{25}$

For each of your tessellations, be sure to have (1) A creative title, (2) What your original shape was and (3) What kind of a tessellation it is. (For example, "Clowns Stomping Spectators", Isos. Triangle, Reflection)

Imagination

It is helpful to have a good imagination when creating animated shapes. Certainly Escher had a great imagination. So did the student who saw at least six different images in the shape below, which he created by rotation modifications.



duck



bird



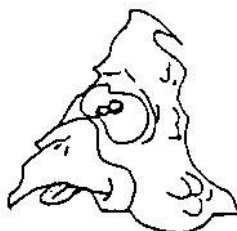
mutant horse



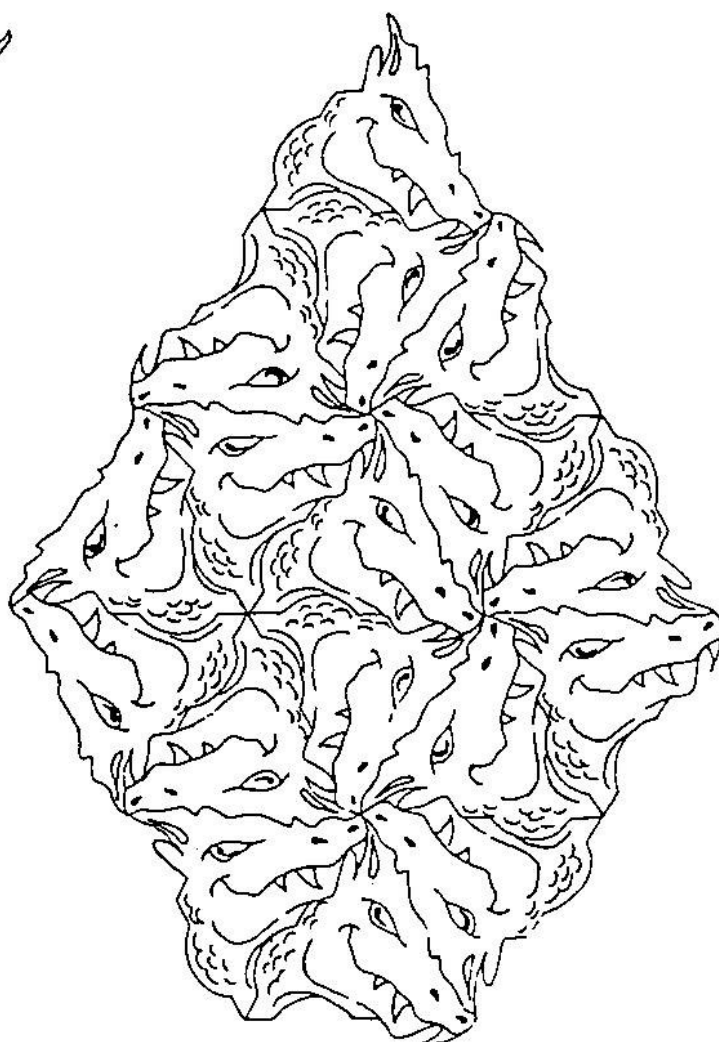
goblin



dragon



cuckoo



Art by Steve Dawson